# STUDENTS' CONCEPT UNDERSTANDING OF DERIVATIVE FUNCTION BASED ON APOS THEORY

**Muchamad Subali Noto[1*], Trusti Hapsari[2], Amalia Fitria Wardhani[3]**
[1,2]Department of Mathematics Education, Universitas Swadaya Gunung Jati, Cirebon, Indonesia; msnoto@ugj.ac.id
[3]Sekolah Menengah Pertama Negeri 7 Cirebon, Indonesia

**Abstract**
Concept understanding of the derivative of algebraic functions can be analyzed through genetic decomposition analysis as an operationalization of APOS Theory. APOS Theory is a mathematics learning theory that assumes that a mathematical concept can be constructed through the action-process-object stage. The purpose of this study was to describe students' concept understanding of function derivatives based on APOS theory for high, medium, and low ability categories. This research used descriptive qualitative research method. The subject of this research was 11th Grade consisting of thirty students. The research data were collected through tests and interviews. The data were analyzed descriptively qualitatively. The result show that, high ability category students have a good understanding and have reached the schema stage, medium and low ability categories have less understanding, the medium category only reaches the process stage towards the object and the low category is only at the action stage but one of the subjects is already at the process stage. After getting the results, it can be used as evaluation material if the level of understanding of students' concepts is known so that it is easier to overcome students' problems regarding their understanding.
**Keywords:** Concept Understanding; Derivatives of Algebraic Functions; APOS theory.

## 1. Introduction

The ability to understand mathematical concepts is one of the most important goals in learning mathematics that students must have (Mawaddah & Maryanti, 2016). This is reinforced by National Council of Teachers of Mathematics (NCTM) which states that concept understanding is a very important aspect in the principles of mathematics learning (Jannah, 2022). According to Ramli & Prabawanto (2020), a good understanding of mathematical concepts will allow students to be able to re-express what has been said to students and use it in various situations, especially those related to solving mathematical problems. In fact, students often lack understanding of concepts (Alan & Afriansyah, 2017). In line with the research of Nuraini et al. (2017), students often focus too much on memorizing formulas and ignore concepts.

According to Syafri (2017), students often ignore definitions, theorems or properties that exist in a mathematical topic, the impact is that students are less able to connect between existing and newly acquired mathematical concepts. For example, regarding the problem of derivatives of trigonometric

functions, when students cannot relate between the concept of trigonometry (concepts that have been obtained) with the definition of derivatives of functions (newly obtained concepts), students will find it difficult to understand the problem so that the solution results are less precise. If the situation where students cannot link the process that has been obtained with the newly obtained one continues, it will result in the shallowness of students' knowledge due to lack of understanding. In fact, concepts in mathematics are related to each other (Novitasari, 2016). Likewise, with the concept of algebraic function derivatives, the concept of algebraic function derivatives is new knowledge for grade XI students.

According to Dwirahayu (2018), the concept of derivative of algebraic function is an abstract concept that only provides the symbol . The concept of function derivative will be used to understand function integral. The concept of function integral is also an abstract concept, therefore students need to have a good understanding of the derivative concept. The difficulties faced by students in answering the material of function derivatives are difficulties in understanding the basic concepts of function derivatives, understanding the problems so that students have difficulty in determining what formulas to use and the lack of students in understanding the question prompts (Anggreini, 2018). Students' understanding of the concept of algebraic function derivatives can be analyzed through a genetic decomposition analysis as an operationalization of APOS theory. MR et al. (2017), said that genetic decomposition analysis is an analysis that focuses on genetic decomposition based on activities, processes, objects, and schemes (APOS) done by each person in solving a problem. Furthermore, an example of genetic decomposition adapted from Arnon et al. (2014), can be seen in Figure 1.

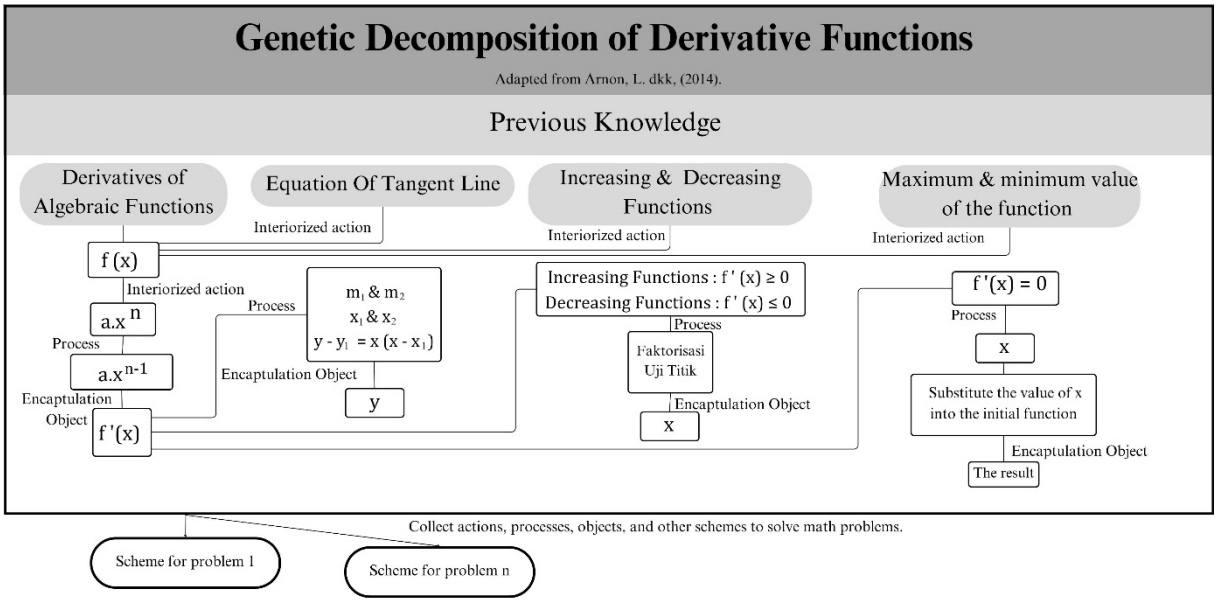


**Figure 1.** Genetic Decomposition of Derivative Algebraic Functions

Figure 1 shows that understanding a mathematical concept or principle begins with manipulating physical or mental objects that have been previously constructed, forming actions. Then, the action is internalized into a process that is encapsulated to form an object. The object can be encapsulated in forming the process again. Cognitive processes such as internalization, encapsulation, and thematization can be well explained through APOS theory. APOS theory is a constructivist theory of student understanding of a concept through the stages of action, process, object and scheme (Anam et al., 2019). APOS theory is useful in understanding how students learn a math topic. One of the topics is in calculus in which there are limits, chain rules, graphical understanding of derivatives, and infinite series (Yerizon, 2011).

From the description above, it can be concluded that APOS Theory can be used as an analytical tool used by researchers to determine students' understanding of calculus topics and the concept of derivative is one of the studies on calculus topics.

## 2. Methods

This research uses a qualitative approach with descriptive method. The subjects in this study were 11th grade high school students obtained from student math ability data. Student math ability data is used to group students based on their abilities. The subjects of this research consisted of 30 students with an average age of 16, comprising 18 female and 12 male students. The

cognitive development level of high school students at this age has reached a point where they can understand problems presented in verbal form and solve them in more effective and complex ways. They also have the opportunity to be trained in solving abstract problems and can develop their own thought patterns in handling a given issue. The initial abilities possessed by students can also influence the learning process of a particular subject. From the total number of subjects, a sample of two students from each category was selected for interviews. The categories are two high ability subjects, two medium ability subjects and two low ability subjects.

The research subjects were selected based on the average final grade of mathematics in the previous semester, based on the test results, and on the basis of suggestions from the mathematics teacher who taught 11th grade students. After the test, the next step is to check the students' answers to categorize students of high, medium and low ability. In determining the category of the test results is done by means of categorization based on the average value and standard deviation. Data collection in this study used test, interview and documentation methods. This research was conducted by giving test questions in the form of description questions with a total of four questions. After that, interviews were conducted with subjects representing high ability, medium ability and low ability. Data analysis in this study using the flow of data analysis activities developed by Miles & Hubermen (2014), namely data reduction, data presentation, and drawing conclusions.

## 3. Results and Discussion

The research results show the recapitulation of the subjects' test results based on ability categories, divided into three levels: high ability, medium ability, and low ability. This categorization is based on the score intervals obtained by the subjects. The following recapitulation of test results based on high, medium and low categories can be seen in table 1.

Table 1. Recapitulation of Subject Test Results by Category

| No. | Category | Grade interval | Number of subjects | Percentage |
|---|---|---|---|---|
| 1. | High ability | $x > 82{,}5$ | 5 | 16.67% |
| 2. | Medium Ability | $45 < x < 82{,}5$ | 16 | 60.00% |
| 3. | Low ability | $x < 45$ | 9 | 23.33% |

Table 1 presents an overview of the proportion of students' abilities in completing the given test, with the majority of students falling into the

medium ability category. Furthermore, each APOS theory has its own characteristics, but researchers here focus on each problem having different stages, for example number one researchers focus on the action stage, number two on the process stage, number three schemes and number four only up to the object. Furthermore, the existence of these stages is intended by researchers to see when students explore the material of function derivatives. So that from the results of the research that has been done, the results show that these students have different understandings, namely:

**Concept Understanding Based on APOS Theory of High Ability Category Students on Problem Number 1 in terms of action.**

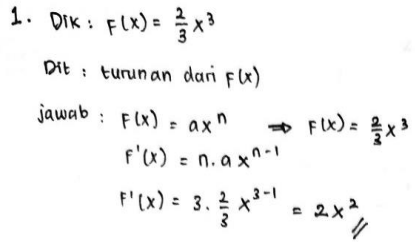


**Figure 2** Answers of high ability category students in question no. 1

Figure 2 shows that the subject can solve problem number 1 by using the basic rules of derivatives and writing symbols correctly. In addition, writing the known and questioned in the problem. Based on this, the subject is already in the action stage. Furthermore, it can be clarified by interview as follows:

P : What do you know about functions and what is the derivative of a function?

S1T : The function usually contains a variable and has a value if a value is entered, what is it, the variable is replaced with a value. Well, if the function's value is like dy/dx, then if so, the maximum and minimum value applications are usually made and then only algebra is taught.

P : List the known functions from question number 1!

S1T : Given the function $f(x) = 2x^3$.

P : What basic derivative rule formula can be used to answer question number 1?

S1T : The formula originally had the function . Then, the power is multiplied by the coefficient of x so n times a, a being the coefficient of x. Then x is fixed and multiplied by n - 1. So, the value of a is 1. Sorry, the value of a is 2/3. The value of n is 3.$f(x) = a.x^n$.

P : Is there enough information to answer the problem asked in question number 1? Give your reasoning?

S1T : Yes. The first reason is that the function is known, then the rule has already been learned, so it's just a matter of applying it.

P : Explain how to apply the known function with the basic rule formula you used in number 1?

S1T : So, we have a is 2/3 and n is 3. Then according to the rule, n times a. The n is 3 times 2/3. We can simplify it to just 2. Then x is the power of 3 because the power is 3 - 1 so the power is 2. So, $2x^2$.

From the results of the interview with the subject, it can be identified that S1T can explain the function, derivative function and the difference. And can explain how the steps in solving problem number 1 which according to the subject according to what he has learned from his math teacher. However, when the interviewer asked what the value of a was, the subject here had answered incorrectly and then a few seconds later the subject realized his mistake and answered correctly the value of a, so S1T already had a good understanding of problem number 1 and was already at the action stage. In line with Rosali (2019), the subject can re-present the rank rule by applying the formula for the derivative rule of the algebraic function in determining the derivative of the algebraic function, so the subject has an understanding of the concept at the action stage in understanding the derivative concept.

**Concept Understanding Based on APOS Theory of Medium Ability Category Students on Problem Number 2 in terms of Process**

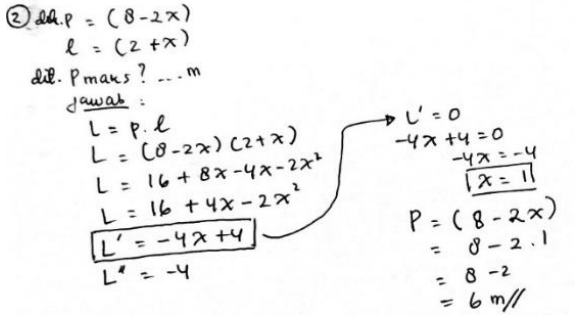


**Figure 3** S2S's answer to question number 2

Figure 3 shows that S2S was able to solve problem number 2 using the derived function in determining the maximum area of the rectangle by applying the right steps. However, in the known step S2S did not write the unit and was wrong in finding the length for the maximum area. Furthermore, in determining the derivative, S2S can perform procedural activities correctly. Therefore, S2S is already in the process stage. This can be clarified with the following interview:

P : State what is known and asked from question number 2?

S2S : the length is (8-2x) m the width is (2+x) m. it is asked that for the maximum area of the rectangle, its length is how many meters.

P : What math concept will be used to solve problem number 2?

S2S : Our concept is to find the area first, we know l accent so we will get the maximum later, it will be related to the stationary so f accent x is equal to zero so L' = 0 get the x then we can find the length by entering the x.

P : Is there enough information to answer the problem asked in question number 2? Give your reasoning?
S2S : Enough, because we already know the length and width we just need to find the area.
P : How to determine the relationship between variables?
S2S : I'm Confused.
P : Can you describe the procedure/steps you used in solving the problem?
S2S : Find the area first, we find the length by finding the accent area then L accent equals zero, then x is found so 1 then the length can be found by entering x. so 8 minus 2 times 1 so the length equals 6.
P : What arithmetic operations were used to solve problem number 2?
S2S : I'm Confused.

From the results of the interview with the subject, it can be identified that S2S can explain the mathematical concepts used, why the information is sufficient, and use any operations. But S2S could not answer about the relationship between the variables. Based on the results of the written and oral answers to question number 2 in determining the length value so that the maximum area of rice fields, the subject has a good understanding and is already at the process stage. This is in line with Rosali (2019), when the subject can choose, utilize, and use procedures in determining the maximum value of the derivative of an algebraic function appropriately, the subject already has an understanding of the concept at the action and process stages in understanding the derivative concept.

**Concept Understanding Based on APOS Theory of Low Ability Category Students on Problem Number 3 in terms of Scheme.**

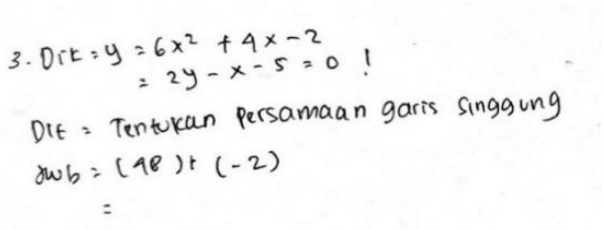


**Figure 4** S3R's answer to question number 3

Figure 4 shows that S3R is unable to solve problem number 3. Because it does not solve the problem, it means that in determining the equation of the tangent line S3R cannot perform procedural activities. Then the scheme stage is not fulfilled by S3R. This can be clarified by interview with S5R as follows:

P : What is known and asked about question number 3?
S3R : We know that equals 2y-x-5=0.$y = 6x^2 + 4x - 2$
P : What mathematical concepts are used to solve problem number 3?
S3R : No idea
P : Is there enough information to answer the problem asked in question number 3? Give your reasoning!

S3R : I forgot how, so I answered like this.
P : Can you describe the steps to solve problem number 3?
S3R : The steps are to find 6x first.
P : Mention the value you get from each step - the solution step! Such as the value of the gradient of the line, the gradient of the curve, the tangent point, and the value of the equation of the tangent line.
S3R : I don't know

From the results of the interview with the subject, it can be identified that S3R cannot explain why the information is sufficient, how the steps in solving problem number 3 and cannot explain what mathematical concepts are used, besides that S3R did not complete his answer only wrote the first step known and asked, meaning that the subject had difficulty in solving problem number 3. In line with research by Sholihah & Mubarok (2016), Based on the results of the written and oral answers to question number 3 in finding the equation of perpendicular tangent S3R has less understanding and is only in the action stage. This is in line with research by Febriana & Budiarto (2013), When the subject does not have the ability to solve the problem, it can be said that all the criteria for the scheme stage are not met by the subject.

**Concept Understanding Based on APOS Theory of Low Ability Category Students on Problem Number 4 in terms of Objects**

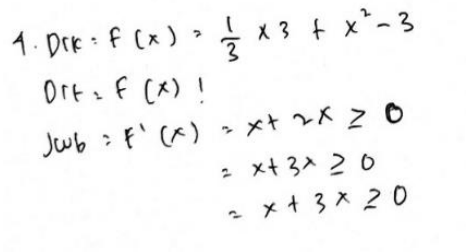


**Figure 5** S4R's answer to question number 4

Figure 5 shows that S4R was unable to solve problem number 4, could not use the procedure (ascending function requirement) in solving the problem correctly, namely there was a mistake in the symbol of the ascending function requirement which should be more than (>) the subject wrote more than equal to (≥), so the object stage was not fulfilled by S4R and can be clarified by an interview with S4S as follows:

P : What is known and asked in question number 4?
S4R : It is known that The question is f(x)$f(x) = \frac{1}{3}x^3 + x^2 - 3$ .
P : List the conditions of an ascending function!
S4R : I don't know, it's random
P : How do I apply the problem to the increasing function condition?

S4R : I don’t remember.
P : Can you describe the steps to answer question number 4?
S4R : I answered that there are 2 of these, so I chose 3.
P : Why did you answer ≥?
S4R : Because it is more than equal to 3.
P : What arithmetic operations did you use to solve problem number 4?
S4R : I don't know.
P : How to determine the test value of x?
S4R : No idea.
P : What concept do you relate to question number 4?
S4R : I don't understand what concept I'm using.

From the results of the interview with the subject, it can be identified that S3R cannot explain the mathematical concepts used, why the information is sufficient and use any operations. And can not explain how the steps in solving problem number 4 and use the point test. However, S3R wrote down the derivative process even though according to S3R the answer was inconsequential and there was a symbol error in the terms of the function up, meaning that the subject had difficulty solving problem number 4. In line with research by Ningsih (2016), This shows that the ability to represent a mathematical concept in the form of other mathematical representations, such as graphics is included in the very low category. So, S3R has less understanding and is only at the action stage. Based on the results of the written and oral answers to question number 4 in determining the interval value of the ascending function, the subject has a lack of understanding and is only at the action stage. In line with research by Khusniah & Maghfiroh (2022), There is an error in the use of symbols where the subject does not understand the concept of using symbols correctly, meaning that the subject makes conceptual errors at the action stage in APOS theory and is not fulfilled at the object stage.

Based on the explanation above, the percentage of students' conceptions and recapitulation of APOS theory experienced by students on the material of function derivatives can be seen in Table 4.3.

**Table 4.3 Percentage of Students' Conceptions of Mathematics Understanding on APOS Theory**

| Question Number | Category of Math Comprehension | Descriptor | Percentage |
|---|---|---|---|
| **1** | Action Design | Did not solve the question. | 00.00%<br>16.67% |

| | | | |
|---|---|---|---|
| | | Answered but got the operation wrong and did not write the formula for the basic rule of derivatives.<br>Use the correct operating procedure and write the formula of the basic rule of derivatives. | 83.33% |
| **2** | Process Conception | No answer or errors in identifying.<br>Can identify but not explain.<br>Can identify and can explain correctly. | 16.67%<br>16.67%<br>66.66% |
| **3** | Object Conception | Presents an answer but it is not correct<br>Presents the answer correctly | 66.66%<br>33.34% |
| **4** | Scheme Conception | Answered but the reason is not relevant<br>Answer with logical reasoning | 66.66%<br>33.34% |

Table 4.3 shows that the subject's action conception in problem number one is perfect. Furthermore, the subject's process conception in question number two is that not all subjects can identify and explain correctly. Problem number three regarding object conception only a few subjects can present their answers correctly. Finally, question number 4 regarding the conception of the scheme where there are several subjects who answer logically. In line with the research of Umar & Afrilianto (2021), There are students who present concepts well in representational form, but there are students who still make mistakes when presenting them.

**Table 4.4 Profile of Students' Concept Understanding Ability based on APOS Theory**

| Question number | APOS | High Ability Students | Medium Ability Students | Low Ability Students |
|---|---|---|---|---|
| **1** | **Action** | Able to explain functions and derivatives of functions and solve problems with correct results and can write the basic rule formula for derivatives of functions. | Able to solve with the correct result, but unable to explain function and function derivative. Also, unable to write down the basic rule formula of the derivative of the function. | Able to solve with the correct result, but unable to explain function, function derivative and the difference. And can write the formula for the basic rule of the derivative of the function. |

| | | | | |
|---|---|---|---|---|
| **2** | **Process** | Able to identify problems in finding the maximum value and can explain the stages or steps of solving correctly. | Able to identify the problem in finding the maximum value but cannot explain the stages or steps of the solution correctly. | Not able to solve the problem means that you cannot identify the problem in finding the maximum value but cannot explain the stages or steps of solving correctly. |
| **3** | **Object** | Able to answer correctly and can relate the concepts used in solving the ascending function problem. | Able to answer correctly. However, unable to relate the concepts used in solving the ascending function problem. | Unable to complete the answer of the ascending function problem and there is a conceptual error where there is an error in the use of symbols and conditions for the ascending function. |
| **4** | **Scheme** | Able to solve and present the answer of the perpendicular tangent equation problem correctly and use relevant reasons. | Unable to solve and present the answer of the perpendicular tangent equation correctly and the reasoning answer is irrelevant. | Unable to solve and present the answer of the perpendicular tangent equation problem and irrelevant reasoning answers. |

Table 4.3 shows that students who have a high level of ability also have a good understanding of the concept and are already at the schema stage, moderate student ability has less concept understanding and up to the object stage only, and low student ability has less concept understanding and is only at the process stage. In line with the research of Afgani et al. (2017), no student reaches the conception of the process or object or scheme perfectly. One of the factors causing this is the method applied in previous learning activities, this method does not provide extensive opportunities for students to construct mathematical understanding (Aydin & Mutlu, 2013; Mrdja et al., 2015).

Based on the results of the analysis that has been carried out through student test results, researchers get findings regarding the level of understanding of student concepts. In line with the findings of Altieri & Schirmer (2019), shows

that the test instrument is very suitable as a measure of the depth and learning outcomes of students. Students who became research subjects were representatives of high, medium, and low ability students. From the results of the research that has been carried out, the results show that these students have different understandings, namely:

(a) High ability subject category

At the number one action stage based on mental construction action has a good performance where it is able to provide systematic and directed answers in answering questions. In line with the research of Listiawati et al. (2023), The subject's understanding has been in the mental structure of action when the subject can determine the derivative of the function using the derivative rule; At the process stage number two the subject can determine the length of the rice field so that the maximum area is correct, so he already has a good understanding and is already at the process stage. In line with research of Sholihah & Mubarok (2016), The subject is able to use, utilize and select certain procedures and solve the problem appropriately according to the procedure so that it is already at the process stage; At the object stage number four in determining the interval value of the ascending function already has a good understanding and is already at the object stage. This is in line with the research of Rahmawati et al (2020), At the object stage, the subject can mention the properties of the function, explain what the function is and how it relates to other concepts or other materials. When the subject has used all the information in the action and process stages to sketch the line, it is in the object stage (Mulyono, 2012); and at the scheme stage number three in determining the equation of the perpendicular tangent line the subject already has a good understanding and is already at the scheme stage. This is in line with the research of Anam et al. (2019), the subject is in the schema stage because he can give the final answer correctly and explain his answer again.

(b) Medium ability subject category

At the action stage number one there is a subject who ignores the question command where the subject does not write the basic formula for the derivative of the function. In line with the opinion of Effendi et al. (2022), The subject ignored the instructions written in the problem. The subject wrote what the subject knew about the problem but only part of the information obtained was written in his worksheet so that the subject was in the action stage (Pangasta et al. 2022). Therefore, it has less understanding but is already at the action stage; At the process stage number two the subject can explain the stages of completion correctly. In line with the research of Rosali (2019), When the subject can choose, utilize, and use procedures or stages in determining the

maximum value of the derivative of an algebraic function appropriately, the subject already has an understanding of the concept at the action and process stages in understanding the concept of derivatives; At the object stage number four is unable to reflect on the action applied to a particular process in answering questions contained in the test sheet, meaning that the object's mental construction performance is poor. In line with the opinion of MR et al. (2017), The dominant factor causing this poor picture of Object mental construction is the poor performance of the mental construction of Action and Process of the subject Therefore, students must fully understand all the prerequisite material related to the ascending function material. And at the number three scheme stage, namely the mental construction of the scheme is the highest level of mental construction in APOS Theory where the mental construction of the scheme has poor performance. In line with MR et al. (2017), the dominant factor causing the poor performance of the mental construction of the Scheme is the poor performance of the mental construction of the Action, Process and Object of the subject.

(c) Low ability subject category

At the action stage number one the subject cannot write down the basic rule formula of the derivative and cannot explain the derivative of the function. In line with the research of Andriani et al. (2017), the lack of understanding of student concepts or students' inability to define concepts is partly because students are accustomed to working on problems instantly without knowing the definition clearly. And in defining verbal and writing is still lacking (Arsalan et al., 2019).

In the process stage number two, there are subjects who do not solve the problem, meaning that the subject cannot understand the stages of completion, the formula to be used, and errors in operations, meaning that the subject has difficulty in determining the strategy or stages to be used. According to Oktac (2019), when processes and objects appear in the same problem situation, students may have difficulty identifying strategies to deal with them. Based on this, the subject could not master the pre-requisite material. According to Susilo et al. (2019), said that students must be able to master the prerequisite material in order to understand the next material.

Furthermore, at the object stage number four cannot solve it. However, the subject only writes what is known and cannot connect the information in the problem in solving it or cannot explain linking existing concepts. This is in line with the research of Sholihah & Subroto (2018), in their research found that students could not connect the information in the problem. Suendarti & Liberna (2021), students are only able to answer what is known and asked in the problem.

At the number three scheme stage, there are subjects who cannot complete the answer, meaning that the subject cannot relate various existing concepts and cannot solve the problem using actions, processes, objects and other schemes that students have. In line with the research of Zahro et al. (2021), low ability subjects have difficulty developing mental constructions until they do not reach the schema stage. One way for students to understand mathematics is to encourage students to find representations as a way of thinking about understanding mathematical ideas.

According to Umar & Afrilianto (2021), there are students who present concepts well in representational form, but there are students who still make mistakes when presenting them. Andriani et al. (2017), said that students still do not understand how to apply concepts to the problems given and also students' understanding of the problem is still lacking. In line with research conducted by Suraji et al. (2018), said that some students were still confused about understanding the problem to be converted into a mathematical model, causing students to misinterpret the questions asked. Also, students have difficulty in using and utilizing and selecting certain procedures or procedural operations. In this indicator, the obstacles experienced by students are that students cannot solve problems when the problems given are different from the examples given by the teacher. This is in line with the research of Sari et al. (2018) which states that students can only work on the same problems that their teachers give them.

Based on genetic decomposition which includes Action, Process, Object, and Schema, it shows that there are several factors that cause students' lack of concept understanding in solving derivative problems of algebraic functions, namely:

(1) Students do not understand the concept of function derivatives. When solving equations of tangent lines, increasing and decreasing functions, as well as maximum and minimum functions, it should be preceded by mastering and understanding the concept. Students tend to memorize the formula and the solution process rather than mastering and understanding the concept. This is in line with the statement of Lestari (2015) which states that students prefer to memorize formulas and procedures for solving problems rather than understanding existing concepts, if students do not remember the memorized formulas and procedures then students will not be able to solve the problem. In addition, Rengkung et al. (2022) stated that the cause of students making mistakes in solving problems is because students do not understand the concept of the material.

(2) Students are less careful in solving problems. Every stage in solving problems requires accuracy so that it can minimize the mistakes made. This is in line with the statement of Junaedi (2012) that students are less careful in

solving problems, lack of practice problems and haste in solving problems can cause low student ability.
(3) The skills possessed by students in solving problems are still lacking, because each problem has a different solution process, so it requires skills in solving it.
(4) Lack of student understanding of prerequisite material. Prerequisite material is material that must be mastered before learning the next material. This is in line with the statement of Israhayu et al. (2021) that the cause of students making mistakes in solving math problems is the lack of student knowledge in understanding prerequisite material. The solution to overcome the problems experienced by students in understanding the concept is:

1. Students are more active and practice more function derivative problems and deepen the material from a conceptual point of view so that students do not have difficulty when facing problems that are different from the examples given by the teacher.
2. Students more often ask or discuss with teachers or friends about the difficulties they face.
3. In learning, students should try to understand the meaning of a symbol, definition, theorem, or properties contained in mathematics, so that they can feel that it will make it easier for them to solve a problem.

Teachers can emphasize students' mature conceptual understanding by using explanations or words that are easily understood by students with the aim of reducing the abstractness of material about the derivative of functions. In addition, in discussing material or problems, it should be combined with prerequisite material such as material about functions, algebra, derivative concepts, square roots, and other related concepts. As well as providing more and more diverse problems, especially problems about the derivative of functions whose solutions allow students to find basic principles, make connections with the definition of derivatives (intuitively), theorems (properties) of function limits, and other properties of function derivatives.

## Conclusion

Understanding the concept of the derivative of algebraic functions based on apos theory in high school students in class XI in the high ability category can have a good understanding and have fulfilled all stages of apos, the medium ability category has less understanding and only reaches the stage of the process towards the object, and the low ability category has less understanding and only at the action stage towards the process.

## References

Afgani, M. W., Suryadi, D., & Dahlan, J. A. (2017). Analysis of Undergraduate

Students' Mathematical Understanding Ability of the Limit of FunctionBased on Theory Perspecive. *Journal of Physics: Conference Series*, *895*. https://doi.org/10.1088/1742-6596/895/1/012056

Alan, U. F., & Afriansyah, E. A. (2017). Students' mathematical comprehension ability through auditory intellectualy repetition and problem-based learning models. *Journal of Mathematics Education*, *11*(1), 67–78. https://doi.org/http://dx.doi.org/10.22342/jpm.11.1.3890.67-78

Altieri, M., & Schirmer, E. (2019). Learning The Concept Of Eigenvalues And Eigenvectors: A Comparative Analysis Of Achieved Concept Construction In Linear Algebra Using APOS Theory Among Students From Different Educational Backgrounds. *ZDM Mathematics Education*, 1125–1140. https://doi.org/https://doi.org/10.1007/s11858-019-01074-4

Anam, A., Anam, A. C., Juniati, D., & Wijayanti, P. (2019). Understanding the quadrilateral concept of junior high school students based on APOS theory in terms of differences in cognitive styles. *Mathematics, Informatics, Science, and Education International Conference (MISEIC)*, 75–78. https://doi.org/https://doi.org/10.2991/miseic-19.2019.24

Andriani, T., Suastika, I. K., & Sesanti, N. R. (2017). Analysis of Errors in Mathematical Concepts of Students in Solving Trigonometry Problems Class X TKJ SMKN 1 Gempol Academic Year 2016/2017. *Pi: Mathematics Education Journal*, *1*(1), 34–39. https://doi.org/https://doi.org/10.21067/pmej.v1i1.1998

Anggreini, D. D. (2018). *Analysis of Students' Understanding of Functional Derivatives Based on Apos Theory in Students of Class XI Iis U Man 1 Tulungagung Academic Year 2017/2018*. UIN SATU Tulungagung.

Arnon, I., Cottrill, J., Dubinsky, E., Oktac, A., Fuentes, S. R., Trigueros, M., & Weller, K. (2014). *APOS Theory: A Framework for Research and Curriculum Development in Mathematics Education*. Springer. https://doi.org/10.1007/978-1-4614-7966-6

Arsalan, A. I., Rosita, C. D., & Dewi, I. L. K. (2019). Design of Teaching Materials on Composition Materials and Inverse Functions based on Mathematical Comprehension Ability. *In Proceedings of the National Seminar on Mathematics Education (SNPM)*, *1*(1), 51–61.

Aydin, S., & Mutlu, C. (2013). Students Understanding Of The Concept Of Limit Of A Function In Vocational High School Mathematic. *TOJSAT*, *3*(1), 145–152.

Dian Novitasari. (2016). The Effect of the Use of Interactive Multimedia on Students' Ability to Understand Mathematical Concepts. *Fibonacci*, *2*(2). https://doi.org/https://doi.org/1024853/fbc.2.2.8-18

Dwirahayu, G. (2018). Analysis of Students' Algebraic Thinking Ability based on Misconceptions. *Institutional Repository of UIN Syarif Hidayatullah Jakarta*.

Effendi, A., Nuryani, V., & Fatimah, A. T. (2022). Analysis of Junior High School Students' Problem-Solving Skills on the Subject of Triangles. *J-KIP: Journal of Teacher Training and Education*, *3*(1), 103–110.

Febriana, C., & Budiarto, M. T. (2013). The Profile of High School Students' Ability to Solve Quadratic Function Problems based on APOS Theory is reviewed from the Difference in Mathematical Ability. *MATHEdunesa*, *2*(3). https://doi.org/https://doi.org/10.26740/matheduneSa.v2n3.p%25p

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). Qualitative data analysis: A methods sourcebook (3rd ed.). SAGE Publications.

Israhayu, N., Syamsuri, S., & Pamungkas, A. S. (2021). Construction of Arithmetic and Geometry Bariisan Concepts for High School Students based on APOS Theory. *Wilangan: Journal of Innovation and Research in Mathematics Education*, *2*(2). https://doi.org/http://dx.doi.org/10.56704/jirpm.v2i2.11649

Jannah, F. L. (2022). *Analysis of Students' Learning Obstacles in Approaching the Concept of Limas' Surface Area*. Gunung Jati Independent University.

Junaedi, I. (2012). Types of Student Errors in Solving Analytical Geometry Problems based on Newman's Error Analysis (NEA). *Kreano: Journal of Creative-Innovative Mathematics*, *3*(2). https://doi.org/https://doi.org/10.15294/kreano.v3i2.2872

Khusniah, R., & Maghfiroh, R. El. (2022). Exploration of Student Errors in Derivative and Integral Materials based on APOS Theory. *Journal of Citizenship*, *6*(3). https://doi.org/https://doi.org/10.31316/jk.v6i3.4099

Lestari, K. E. (2015). Application of the M-APOS Learning Model to Improve Junior High School Students' Mathematical Solving Skills. *Judika: Unsika Education Journal*, *3*(1). https://doi.org/https://doi.org/10.35706/judika.v3i1.200

Mawaddah, S., & Maryanti, R. (2016). The Ability to Understand Mathematical Concepts of Junior High School Students in Learning using the Discovery Learning Model. *EDU-MAT: Journal of Mathematics Education*, *4*(1). https://doi.org/http://dx.doi.org/10.20527/edumat.v4i1.2292

MR, A., Widada, W., & Abdullah, M. I. (2017). Mathematical Problem Solving Ability Reviewed Based on Genetic Decomposition in Grade VII Students of SMPN 2 Pondok Kelapa, Central Bengkulu Regency. *MUST: Journal of Mathematics Education, Science and Technology*, *2*(1), 20–42. https://doi.org/http://dx.doi.org/10.30651/must.v2i1.402

Mrdja, M., Romano, D. A., & Zubac, M. (2015). Analysis Of Students' Mental Structures When Incorrectly Calculating The Limit Of Functions. *IMVJ: Open Mathematical Education Notes*, *5*(2), 101–113.

Mulyono. (2012). Field Dependent Students' Understanding in Reconstructing the Concept of Functional Graphs. *Kreano: Journal of Creative-Innovative Mathematics*, *3*(1). https://doi.org/https://doi.org/10.15294/kreano.v3i1.2612

Ningsih, Y. L. (2016). The ability to understand students' mathematical concepts through the application of Student Activity Sheets (LAM) based on APOS Theory on Derivative Materials. *Edumatica : Mathematics Education Learning and Teaching*, *6*(1). https://doi.org/https://doi.org/10.22437/edumatica.v6i01.2994

Nuraini, N. L. S., Suartono, & Yuniawatika. (2017). Student Errors in Addition and Subtraction Operations of Fractions in Grade VI of Elementary School. *Elementary School: A Study of Educational Theory and Practice*, *2*(25), 168–175.

Oktac, A. (2019). Mental constructions in linear algebra. *ZDM Matematics Education*, 1043–1054. https://doi.org/https://doi.org/10.1007/s!1858-019-01037-9

Pangasta, D. G., Cahyo, H., & Jamil, A. F. (2022). Scaffolding analysis in solving mathematical problems is reviewed from the APOS framework. *Proceedings of the National Seminar on Mathematics and Its Learning*, *1*.

Rahmawati, M., Uswatun, D. A., & Maula, L. H. (2020). Analysis of students'

understanding of mathematical concepts based on apos theory through online-based open-ended questions in elementary high school grades. *Didactic: Scientific Journal of PGSD STKIP Subang*, *6*(1), 155–165. https://doi.org/https://doi.org/10.36989/didaktik.v6i1.122

Ramli, & Prabawanto, S. (2020). Difficulties and Learning Obstacles for Students in Solving Mathematical Problems based on Understanding Mathematical Concepts. *JURING: Journal For Researc in Mathematics Learning*, *3*(3). https://doi.org/http//dx.doi.org/10.24014/juring.v3i3.9999

Rengkung, A. L., Pesik, A., & Pitoy, C. (2022). Analysis of Student Errors in Solving Problems in the Operation Material of the Form of ALjabar. *ADIBA: Journal Of Education*, *2*(2).

Rosali, D. F. (2019). *Description of the Ability to Understand Derivative Concepts based on APOS Theory in Grade XII MIA-1 SMAN 2 Makassar Students*. Makassar State University.

Sari, M., Habibi, M., & Putri, R. (2018). Influence of the Think-Pairs-Share Type Cooperative Learning Model in Mathematics Learning on the Ability to Understand Mathematical Concepts and Character Development of High School Students in Sungai TFull City. *Edunatika: Journal of Mathematics Education Research*, *1*(1). https://doi.org/https://doi.org/10.32939/ejrpm.v1i1.221

Sholihah, U., & Mubarok, D. A. (2016). Analysis of Infinite Integral Understanding based on APOS Theory in Mathematics Tadris Students of IAIN Tulungagung. *Scholar: Journal of Education and Society*, *14*(1). https://doi.org/10.21154/cendekia.v14i1.620

Sholihah, W., & Subroto, T. (2018). Analysis of Learning Barriers to Trigonometry Materials in Students' Mathematical Comprehension Ability. *IndoMath: Indonesia Mathematic Education*, *1*(2). https://doi.org/https:/doi.org/10.30738/indomath.v1i2.2624

Suendarti, M., & Liberna, H. (2021). Analysis of the Understanding of the Concept of Trigonometry Comparison in High School Students. *JNPM : National Journal of Mathematics Education*, *5*(2). https://doi.org/http://dx.doi.org/10.33603/jnpm.v5i2.4917

Suraji, S., Maimunah, M., & Saragih, S. (2018). Analysis of Mathematical Concept Comprehension Ability and Mathematical Problem Solving Ability of Junior High School Students on SPLDV Material. *Journal of Mathematics Education*, *4*(1). https://doi.org/https://dx.doi.org/10.24014/sjme.v4i1.5057

Susilo, B. E., Darhim, D., & Prabawanto, S. (2019). Student Learning Difficulties on Integral Application Materials for Regional Areas in the Perspective of Mathematical Disposition. *Kreano: Journal of Creative-Innovative Mathematics*, *10*(1), 86–93. https://doi.org/https://doi.org/10.15294/kreano.v10i1.19373

Syafri, F. S. (2017). Understanding mathematics in the study of apos theory (action, process, object, and schema). *At-Ta'lim: Islamic Educational Information Media*, *15*(2), 458–477. https://doi.org/http://dx.doi.org/10.29300/attalim.v15i2.536

Umar, N. F., & Afrilianto, M. (2021). Improving Junior High School Students' Conceptual Understanding of Relationship and Function Materials with a Scientific Approach. *JPMI: Journal of Innovative Mathematics Learning*, *4*(2).

https://doi.org/https://doi.org/10.22460/jpmi.v4i2.p%25p

Yerizon. (2011). Improving Students' Proof Ability and Independence in Learning Mathematics through the M-APOS Approach. In *repository uny*. University of Education Indonesia.

Zahro, F., Hidayati, Y., Qomaria, N., & Ahied, M. (2021). Description of Student Understanding based on APOS Theory on Temperature and Heat Subjects. *Journal of Natural Science Educational Research*, *4*(1). https://doi.org/https://doi.org/10.21107/nser.v4i1.8484